

Exam. Code : 211001

Subject Code : 5476

M.Sc. Mathematics 1st Semester

DIFFERENTIAL EQUATIONS

Paper—MATH-555

Time Allowed—3 Hours] [Maximum Marks—100

**Note** :—Attempt **TWO** questions from each unit. All questions carry equal marks.

**UNIT—I**

1. State and prove existence and uniqueness theorem for the solution of the equation :

$$\frac{dy}{dx} = f(x, y), f(x_0) = y_0.$$

2. Prove that the continuity of  $f(x, y)$  is not enough to guarantee the uniqueness of the solution of the initial value problem :

$$\frac{dy}{dx} = f(x, y) = \sqrt{|y|}; y(0) = 0.$$

3. State and prove the necessary and sufficient condition for a general second order homogeneous linear differential equation to be self adjoint.
4. Obtain the first four successive approximation for the solution of  $y' = y^2 - x^2$ ,  $x = 0$ ,  $y = 1$ .

## UNIT—II

5. Using convolution theorem find the inverse Laplace transform of  $\frac{1}{S^2(S+1)^2}$ .
6. Evaluate

$$\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t} dt$$

7. Find  $L\{f(t)\}$  where  $f(t) = \begin{cases} \cos t & 0 < t < 2\pi \\ 0 & t > 2\pi \end{cases}$ .
8. Find the solution of Initial value problem  $ty'' + 2ty' + 2y = 2$ ;  $y(0) = 1$ ,  $y'(0)$  arbitrary.

## UNIT—III

9. State and prove convolution theorem for Fourier transform.
10. Find Fourier transform of

$$f(x) = \begin{cases} a - |x| & ; |x| < a \\ 0 & ; |x| > a \end{cases}$$

Hence evaluate  $\int_0^{\infty} \left( \frac{\sin t}{t} \right)^2 dt$ .

11. Find the Fourier cosine transform of  $e^{-x^2}$ .

12. Use complex form of Fourier transform to show that

$$u \frac{1}{2\sqrt{\pi t}} \int_{-\infty}^{\infty} f(u) \exp\left\{-\frac{(x-u)^2}{4t}\right\} dt \text{ is the solution of the}$$

boundary value problem  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ ,  $-\infty < x < \infty$ ,  $t > 0$ ,

$u = f(x)$ , when  $t = 0$ .

#### UNIT—IV

13. Prove that  $P_n x = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ .

14. Define Bessel function of first kind of order  $p$ . Show that  $\frac{d}{dx} [x^p J_p(kx)] = kx^p J_{p-1}(kx)$ .

15. Show that  $\int_0^{\pi/2} J_1(x \cos \phi) d\phi = \frac{1 - \cos x}{x}$ .

16. Prove that  $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$ .

#### UNIT—V

17. Solve  $\frac{dx}{dt} = 3x + 2y$ ;  $\frac{dy}{dt} = -5x + y$ .

18. Show that if  $q$  is continuous and such that  $q(x) > 0$  for  $x > 0$  and  $k$  is (+) ve constant, then every real solution

of  $\frac{d^2 y}{dx^2} + [q(x) + k^2] y = 0$  has infinite no. of positive zeros.



19. Find the characteristic value and characteristic function of the problem

$$\frac{d}{dx} \left[ (x^2 + 1) \frac{dy}{dx} \right] + \frac{\lambda}{x^2 + 1} y = 0, y(0) = 0, y(1) = 0.$$

20. State and prove Sturm Comparison theorem.